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LRS Bianchi Type-I Universe in *F (T)* Theory of Gravity

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Abstract: We have studied the spatially homogeneous and anisotropic Locally Rotationally Symmetric (LRS) Bianchi type-I universe in *F(T)* theory of gravity. By using a conservation equation, we have discussed some well known *F(T)* models. It is interesting to observe that these *F(T)* gravity models represent the different phases (matter, radiation and dark energy eras) of the universe. An attempt has been made to retain Sharif and Rani's [1] forms of the various quantities. Our results are analogous to the results obtained by Sharif and Rani [1].

Keywords: *F(T)* gravity, LRS Bianchi type-I universe, continuity equation.

1 Introduction

The observations of Supernova type-Ia experiments [2-6], cosmic microwave background (CMB) anisotropies [7-9], large scale structure [10-12] have proposed that a peculiar component, generally known as the dark energy (DE), with high negative pressure is pushing this present cosmic accelerated expansion. This component occupies 68.3% of the present universe dominating the other components of the universe. (i.e., dark matter component (26.8%) and baryonic matter (4.9%) as observed by PLANCK 2013 [13].

The concept of DE was introduced by Einstein himself when he added the cosmological constant in the field equations. In spite of all observational evidences, explaining expansion of universe has been an open challenge in modern physics [14]. The most suitable of the modified theories is *F(R)* theory of gravity which is a function of Ricci scalar *R* in standard Einstein-Hilbert Lagrangian [15-20]. The *F(R)* theory gives cosmic inflation and depicts nature of dark energy with present cosmic acceleration. Another modified theory developed by Harko *et al.* [21] is known as *F(R,T)* which is the generalization of *F(R)* gravity and depends upon coupling of matter and geometry. Here the Lagrangian includes a function of the scalar curvature *R* and the trace of energy momentum tensor *T*. Also, Ferraro and Fiorini [22, 23] have developed *F(T)* theory and solved practical horizontal problem as well as obtained singularity free solutions with positive cosmological constant. This *F(T)* theory of gravity is the generalization of teleparallel gravity where curvature free Weitzenböck connections are used. This theory reduces to general relativity if *F(T)* is replaced by a constant [24, 25]. Although *F(R)* gravity has given many cosmologically important models [26-28], it is rather challenging as its equations are of fourth order. On the other hand, *F(T)* produces equations of order two and gives interesting results [29-33].

Our universe is homogeneous and isotropic on large scale. Result obtained by WMAP data [34-36] shows the existence of an anisotropic phase of the universe which approaches isotropy. The anisotropic model has been gaining interest since then. The Bianchi type models are spatially homogeneous and anisotropic. The most basic anisotropic model i.e. the Bianchi Type I (BI) universe has been studied by several researchers to discuss the effect of anisotropy in several contexts. Kumar and Singh [37, 38] have studied the exact solutions for the Bianchi Type I universe in various theories. Sharif and Waheed [39] investigated exact solutions for anisotropic fluid by considering the LRS Bianchi type-I universe which generalizes the flat FRW universe in the modified theory. Work by many researchers on this model is available using different parameters and different theories [40-46].

We have discussed LRS Bianchi type-I models in *F(T)* gravity. In the second section we have presented some basics of teleparallel gravity and the corresponding field equations for LRS Bianchi Type-I are given in section 3. A detailed construction of *F(T)* gravity models is given using continuity equation in section 4. In the last section, we summarize and conclude the results.

2 *F(T)* Gravity Formalism

Let us present *F(T)* gravity. We introduce the modified teleparallel theory of gravity as well as extension to *F(T)* theory. The Lagrangian density for teleparallel and *F(T)* gravity [25] are respectively given by

$$L_T = -\frac{e}{16\pi G} T \tag{1}$$

$$L_{F(T)} = -\frac{e}{16\pi G} F\left(T\right) \tag{2}$$

where T is the torsion scalar, *F(T)* is a differentiable function of torsion scalar T, G is the gravitational constant and $e = \sqrt{-g}$.

The torsion has the form

$$T = S_\rho^{\ \ \mu\upsilon} T^\rho_{\ \ \mu\upsilon} \tag{3}$$

where $S_\rho^{\ \ \mu\upsilon}$ is the antisymmetric tensor; $T^\rho_{\ \ \mu\upsilon}$ is the torsion tensor which are respectively defined by

$$S_\rho^{\ \ \mu\upsilon} = \frac{1}{2}\left(K^{\mu\upsilon}_{\ \ \ \ \rho} + \delta^\mu_\rho T^{\theta\upsilon}_{\ \ \ \ \theta} - \delta^\upsilon_\rho T^{\theta\mu}_{\ \ \ \ \theta}\right) \tag{4}$$

$$T^\lambda_{\ \ \mu\upsilon} = \Gamma^\rho_{\ \ \mu\upsilon} - \Gamma^\rho_{\ \ \upsilon\mu} = h^\lambda_i \left(\partial_\mu h^i_\upsilon - \partial_\upsilon h^i_\mu\right) \tag{5}$$

where $\Gamma^\rho_{\ \ \upsilon\mu}$ is the Weitzenböck connection.

A discretionary option in choosing the vierbein field related to the metric tensor $g_{\mu\upsilon}$ by the following relation

$$g_{\mu\upsilon} = \eta_{ij}\, h^i_\mu h^i_\upsilon \qquad i, j, \ldots = 0,1,2,3\ ; \quad \mu, \upsilon, \ldots = 0,1,2,3. \tag{6}$$

where the field $h_i\left(x^\mu\right)$ at every point x^μ of manifold are orthonormal basis for the space- time and $e_i e_j = n_{ij}$, where $n_{ij} = diag\left(1,-1,-1,-1\right)$ is the Minkowski metric.

Here, h^i_μ satisfies the properties

$$h^i_\mu h^\mu_j = \delta^i_j \ , \quad h^i_\mu h^\mu_i = \delta^\upsilon_\mu \tag{7}$$

The contorsion tensor $K^{\mu\upsilon}_{\ \ \ \ \rho}$ has the following form

$$K^{\mu\upsilon}_{\ \ \ \ \rho} = -\frac{1}{2}\left(T^{\mu\upsilon}_{\ \ \ \ \rho} - T^{\upsilon\mu}_{\ \ \ \ \rho} - T_\rho^{\ \ \mu\upsilon}\right) \tag{8}$$

where contorsion tensor is the connection of difference between Weitzenböck and Levi-Civita.

Now, we present the field equations of *F(T)* theory of gravity for the action (2) with respect to tetrad field

$$\left[e^{-1}\partial_\mu\left(eS_i^{\ \ \mu\upsilon}\right) - h_i^\lambda T^\rho_{\ \ \mu\lambda} S_\rho^{\ \ \upsilon\mu}\right] F_T + S_i^{\ \ \mu\upsilon}\partial_\mu\left(T\right) F_{TT} + \frac{1}{4} h_i^\upsilon F = \frac{1}{2} k^2 h_i^\rho T_\rho^\upsilon \tag{9}$$

where *F(T)* is the general differentiable function of torsion, $k^2 = 8\pi G$, $F_T = \frac{dF}{dT}$, $S_i^{\ \ \mu} = h_i^\rho S_\rho^{\ \ \mu\upsilon}$.

The energy momentum tensor has the components

$$T_\rho^\upsilon = diag\left(\rho_m, -p_m, -p_m, -p_m\right) \tag{10}$$

where ρ_m and p_m are energy density and pressure of matter inside the universe respectively.

3 The Field Equations

Now we assume the line element for homogenous and anisotropic LRS Bianchi type-I space-time having one transverse direction x and two equal longitudinal directions y and z which are responsible for anisotropic behavior [47]

$$ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)(dy^2 + dz^2) \tag{11}$$

where $A(t)$ and $B(t)$ are the cosmic scale factors.

We obtain the vierbein components by using equation (6) and equation (11) as

$$h^i_\mu = diag(1, A, B, B), \quad h^\mu_i = diag(1, A^{-1}, B^{-1}, B^{-1}) \tag{12}$$

By using equations (4) and (5) in equation (3), the torsion tensor for LRS Bianchi type-I has the form

$$T = -2\left(2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2}\right) \tag{13}$$

The above field equations (9) of *F(T)* theory of gravity, using equations (10)-(13) reduce to the following set of equations for $i = 0 = \upsilon$ and $i = 1 = \upsilon$,

$$F - 4\left(2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2}\right)F_T = 2k^2\rho_m \tag{14}$$

$$4\left(\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} + \frac{\ddot{B}}{B}\right)F_T - 16\frac{\dot{B}}{B}\left[\frac{\dot{B}}{B}\left(\frac{\ddot{A}}{A} - \frac{\dot{A}^2}{A^2}\right) + \left(\frac{\ddot{B}}{B} - \frac{\dot{B}^2}{B^2}\right)\left(\frac{\dot{B}}{B} + \frac{\dot{A}}{A}\right)\right]F_{TT} - F = 2k^2 p_m \tag{15}$$

The conservation equation of energy momentum tensor turns out to be

$$\dot{\rho}_m + \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right)(\rho_m + p_m) = 0 \tag{16}$$

The directional Hubble parameter H_i in the direction of x, y and z axes respectively are defined as

$$H_1 = \frac{\dot{A}}{A}, H_2 = H_3 = \frac{\dot{B}}{B} \tag{17}$$

We have defined the average scale factor R, the Hubble parameter H and the anisotropy parameter Δ for LRS Bianchi type-I by

$$R = \left(AB^2\right)^{1/3} \tag{18}$$

$$H = \frac{1}{3}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) \tag{19}$$

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 \tag{20}$$

For $\Delta = 0$, it is observed that isotropic behavior of the universe is obtained, which depends on the values of unknown scale factors and parameters that are involved in the model [47-49].

Using equations (13) and (19), we get

$$T = -9H^2 + J \quad \text{with} \quad J = \frac{\dot{A}^2}{A^2} + 2\frac{\dot{B}^2}{B^2} \tag{21}$$

which is equal to

$$H = \frac{1}{3}\sqrt{J - T} \tag{22}$$

Now using $F(T) = T$, $F_T = 1$ in equations (14) and (15), we get

$$\rho_m + \rho_T = \frac{1}{2k^2}\left[-4\left(2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2}\right) + T\right] \tag{23}$$

$$p_m + p_T = \frac{1}{2k^2}\left[4\left(\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} + \frac{\ddot{B}}{B}\right) - T\right] \tag{24}$$

where ρ_T and p_T are the torsion contribution to the energy density and pressure given by

$$\rho_T = \frac{1}{2k^2}\left[-4\left(2\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}\right)\left(1-F_T\right)+T-F\right] \tag{25}$$

$$p_T = \frac{1}{2k^2}\left[4\left(\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}+\frac{\ddot{B}}{B}\right)\left(1-F_T\right)+16\frac{\dot{B}}{B}\left\{\frac{\dot{B}}{B}\left(\frac{\ddot{A}}{A}-\frac{\dot{A}^2}{A^2}\right)+\left(\frac{\ddot{B}}{B}-\frac{\dot{B}^2}{B^2}\right)\left(\frac{\dot{B}}{B}+\frac{\dot{A}}{A}\right)\right\}\right. \tag{26}$$

We get the solution from the homogeneous part of equation (14) *i.e.* $\rho = 0$ as

$$F\left(T\right) = \frac{c_0}{\sqrt{T}} \tag{27}$$

where c_0 is the constant of integration.

From this solution, equation (15) takes the form

$$p_m = \frac{1}{2k^2}\left[\frac{6\chi\dot{T}}{T^2}-\frac{3\dot{H}+J+\tau}{T}-\frac{1}{2}\right]\frac{c_0}{\sqrt{T}} \tag{28}$$

where $\chi = \frac{\dot{B}}{B}, \tau = \frac{\dot{B}^2}{B^2}$.

4 Construction of Some *F(T)* Models Using Continuity Equation

In this section by using continuity equation (16), we have constructed the *F(T)* models for different cases of perfect fluid and discussed the values of EoS parameter for non-relativistic matter, radiation and D.E.

Here for LRS Bianchi type-I universe, we use equation [50]

$$\frac{1}{9}\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right)^2 = H_0^2 + \frac{k^2\rho_0}{3AB^2} \tag{29}$$

It implies

$$\left(AB^2\right)^{-1} = \frac{3}{k^2\rho_0}\left(H^2-H_0^2\right) \tag{30}$$

where ρ_0 is the constant of integration and H_0 is the Hubble constant.

Equation (16) can be written in terms of EoS parameter

$$\frac{\dot{\rho}_m}{\rho_m}+3H\left(1+\omega\right)=0 \tag{31}$$

Now we construct the *F(T)* models for the different cases of fluids and their combination which are for relativistic matter, non-relativistic matter and DE era [51].

Case 1: For non-relativistic matter i.e. $\omega = 0$. It represents cold dark matter (CDM) and baryons.

In equation (31), we put $\omega = 0$ and from equation (30), we get

$$\rho_m = \rho_c\left(AB^2\right)^{-1} = \frac{3\rho_c}{k^2\rho_0}\left(H^2-H_0^2\right) \tag{32}$$

since ρ_c is a constant of integration.

Also, equation (32) in the form of torsion scalar is

$$\rho_m = \frac{\rho_c}{3k^2\rho_0}\left(J-9H_0^2-T\right) \tag{33}$$

By substituting the values of ρ_m in equation (14) we get

$$2TF_T + F = \frac{2\rho_c}{3\rho_0}\left(J-9H_0^2-T\right) \tag{34}$$

Solution of the above equation is

$$F\left(T\right)=\frac{2\rho_c}{3\rho_0}\left(J-9H_0^2-T\right) \tag{35}$$

Equation (35) gives a unique solution if the unknown scale factor J is known.
So, here we get the model in terms of torsion and Hubble constant.

Case2: For relativistic matter i.e. $\omega=\frac{1}{3}$ which represents photons and massless neutrinos. It indicates the radiation dominated era of the universe.

Putting $\omega=\frac{1}{3}$ in equation (31) and by using equation (21) and equation (30), we get

$$\rho_m=\frac{\rho_r}{3^{4/3}k^{8/3}\rho_0^{4/3}}\left(J-9H_0^2-T\right)^{4/3} \tag{36}$$

where ρ_r is another constant of integration.

Substituting the values of ρ_m in equation (14), it implies

$$2TF_T+F=\frac{2\rho_r}{3^{4/3}k^{2/3}\rho_0^{4/3}}\left(J-9H_0^2-T\right)^{4/3} \tag{37}$$

Equation (37) has the solution

$$F\left(T\right)=\frac{2\rho_r}{3^{4/3}k^{2/3}\rho_0^{4/3}}\left(J-9H_0^2-T\right)^{4/3} \tag{38}$$

Here also the solution depends upon the values of J, torsion scalar T and Hubble constant.

Case 3: For DE era i.e. $\omega=-1$. This represents the major component of the universe i.e. the DE component which has a large negative pressure.

Now by using $\omega=-1$ in equation (31), we get

$$\rho_m=\rho_d \tag{39}$$

where ρ_d is the constant of integration. From equation (39), equation (14) implies that

$$2TF_T+F=2k^2\rho_d \tag{40}$$

Solution of the above equation is

$$F\left(T\right)=2k^2\rho_d \tag{41}$$

This solution gives the constant model which is consistent with cosmological constant.

Case 4: Combination of Dust fluid and radiation i.e. $\omega=0$ and $\omega=\frac{1}{3}$. Here we take the combination of two different fluids, the dust fluid and the radiations.

By adding equation (33) and equation (36) we get

$$\rho_m=\frac{1}{3k^2\rho_0}\left(J-9H_0^2-T\right)\left[\rho_c+\frac{\rho_r}{3^{1/3}k^{2/3}\rho_0^{1/3}}\left(J-9H_0^2-T\right)^{1/3}\right] \tag{42}$$

Put the value of ρ_m in equation (14), it follows that

$$2TF_T+F=\frac{2}{3\rho_0}\left(J-9H_0^2-T\right)\left[\rho_c+\frac{\rho_r}{3^{1/3}k^{2/3}\rho_0^{1/3}}\left(J-9H_0^2-T\right)^{1/3}\right] \tag{43}$$

We obtain

$$F\left(T\right)=\frac{2}{3\rho_0}\left(J-9H_0^2-T\right)\left[\rho_c+\frac{\rho_r}{3^{1/3}k^{2/3}\rho_0^{1/3}}\left(J-9H_0^2-T\right)^{1/3}\right] \tag{44}$$

Case 5: Combination of Dust fluid and DE i.e. $\omega=0$ and $\omega=-1$, which gives

$$\rho_m = \frac{\rho_c}{3k^2\rho_0}\left(J - 9H_0^2 - T\right) + \rho_d \tag{45}$$

By using equation (45), equation (14) takes the form

$$2TF_T + F = \frac{2\rho_c}{3\rho_0}\left(J - 9H_0^2 - T\right) + 2k^2\rho_d \tag{46}$$

This gives

$$F(T) = \frac{2\rho_c}{3\rho_0}\left(J - 9H_0^2 - T\right) + 2k^2\rho_d \tag{47}$$

Case 6: Combination of DE and Radiation i.e. $\omega = -1$ and $\omega = \frac{1}{3}$.

In this section we consider the combination of EoS parameter for DE and radiation dominated era which yields

$$\rho_m = \rho_d + \frac{\rho_r}{3^{4/3} k^{8/3} \rho_0^{4/3}}\left(J - 9H_0^2 - T\right)^{4/3} \tag{48}$$

Using equation (48) in equation (14), we have

$$2TF_T + F = 2k^2\rho_d + \frac{2\rho_r}{3^{4/3} k^{2/3} \rho_0^{4/3}}\left(J - 9H_0^2 - T\right)^{4/3} \tag{49}$$

It implies the following solution

$$F\left(T\right) = 2k^2\rho_d + \frac{2\rho_r}{3^{4/3} k^{2/3} \rho_0^{4/3}}\left(J - 9H_0^2 - T\right)^{4/3} \tag{50}$$

5 Conclusion

By using continuity equation, some *F(T)* gravity LRS Bianchi type-I models have been constructed. These *F(T)* gravity models represent three different phases such as matter, radiation and DE respectively corresponding to $\omega = 0$, $\omega = 1/3$ and $\omega = -1$. Matter dominated era explains expansion of the universe filled with non-interacting dust particles while radiation dominated era represents early universe filled with radiation. The DE era corresponds to the universe dominated by a strong negative pressure causing late time acceleration. An attempt has been made for the revival of the form used by Sharif & Rani [1]. Our results are analogous to the results obtained by Sharif and Rani [1].

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