



Fig. 1 Numerical solution to Example 5.2

Table 1: Solution errors and computational order of convergence for example 5.1.

h	ω	η	E_∞	E_2	O_∞	O_2
1/10	5	0.8	7.59e-03	5.17e-03	---	---
1/20	5	0.8	4.87e-04	2.92e-04	4.0	4.1
1/40	5	0.8	3.06e-05	1.77e-05	4.0	4.0

Table 2: Solution errors and computational order of convergence for example 5.2.

h	ω	η	Itr	E_∞	E_2	O_∞	O_2
1/10	0	1	58	1.26e-01	6.45e-02	---	---
1/20	0	1	95	8.73e-03	4.07e-03	3.8	4.0
1/40	0	1	841	5.48e-04	2.50e-04	4.0	4.0
1/10	20	0.36	19	1.44e-01	6.81e-02	---	---
1/20	20	0.21	80	9.21e-03	4.16e-03	4.0	4.0
1/40	30	0.10	765	5.57e-04	2.54e-04	4.0	4.0

Table 3: Solution errors and computational order of convergence for example 5.3.

h	ω	η	E_∞	E_2	O_∞	O_2
1/10	10	1	1.95e-04	8.29e-05	---	---
1/20	10	1	2.09e-05	6.72e-06	3.2	3.6
1/40	10	1	1.37e-06	4.14e-07	3.9	4.0

Table 4: Solution errors and computational order of convergence for example 5.4.

h	ω	η	Itr	E_∞	E_2	O_∞	O_2
1/10	0.0001	1.0	178	3.72e-03	2.53e-03	---	---
1/20	0.0001	1.0	609	2.43e-04	1.43e-04	3.9	4.1
1/40	0.0001	1.0	2418	1.52e-05	8.66e-06	4.0	4.0
1/10	300	1.0	103	3.58e-03	2.43e-03	---	---

1/20	300	1.0	439	2.09e-04	1.23e-04	4.1	4.3
1/40	300	1.0	2072	7.86e-06	4.49e-06	4.7	4.8
1/10	300	0.8	98	3.49e-03	2.37e-03	---	---
1/20	300	0.8	413	1.81e-04	1.07e-04	4.3	4.5
1/40	300	0.5	1739	1.09e-05	6.22e-06	4.1	4.1

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